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The Correlation between the Wavelet Base Properties and Image Compression

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Abstract

When wavelet transform is applied to image compression, the chosen wavelet base affects the efficiency of coding and the quality of the reconstructed image, because the property parameters of different wavelet bases are varied, it is very important to research the correlation between the wavelet base properties and image compression. Chosen wavelet bases of one family for the experiments are convenient for analyzing contrastively, and the results have high reliability, and Daubechies wavelet bases with the properties of compactly supported, orthogonality, regularity, vanishing moment are widely used, then the paper chooses Daubechies wavelet bases as the research object, analyzes the correlation between the wavelet base properties and the image compression. The experiment results present the principles of wavelet base choice in image compression.

1. Introduction

Every day, an enormous amount of information is stored, processed, and transmitted, so the storage and communication requirements are immense. Methods of compressing the data prior to storage and/or transmission, especially the image data, are of practical significance and commercial interest.

Wavelet transform has good localization properties both in space and frequency domains, which makes up the deficiency of DCT and Fourier transform, and it is flexible in multi-scale solution of representation of image signals, now wavelet transform has emerged as a promising technique for image compression.

When wavelet transform is applied to image compression, the chosen wavelet base directly affects the efficiency of coding and the quality of the reconstructed image, therefore how to choose wavelet base is a key problem in image compression. Now there are many papers as in [1-3] about wavelet base choice, and the results are varied, they have not analyze the correlation between the wavelet base properties and image compression, so the results can not be regard as the principle of wavelet base choice. This paper chooses Daubechies wavelet bases as in [4-7] as the research object, analyzes the properties of wavelet base and the effects on image compression, and gives the principles of wavelet base choice in image compression.

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2. Wavelet Base Evaluation Standards

2.1. Coding Gain

Coding gain is an important measure of the theoretical performance of orthogonal signal coding schemes as in [1]. It represents the energy tight degree of subbands, and is defined by

$$G=\frac{1}{K}\sum_{k=0}^{K-1}\sigma_k^2 \Big/ \left(\prod_{k=0}^{K-1}\sigma_k^2\right)^{\frac{1}{K}} \tag{1}$$

Where K is the number of subbands after wavelet decomposed, and σ_k^2 is the variance of each frequency subband. Coding gain can be used as an objective standard to evaluate wavelet base. The more coding gain is, the better effect of reconstructed image is.

2.2. PSNR of Reconstructed Image

PSNR (Peak Signal-to-noise Ratio) represents the difference between two images as in [1]. PSNR is the evaluation standard of the reconstructed image quality, and is an important measure of image compression. If the methods used for image compression are same, and only the wavelet bases are changed, PSNR can be used as the evaluation standard of the coding performance of wavelet base. The higher PSNR is, the better the reconstructed image is, and the

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wavelet base is more suitable for image compression. For a gray image, PSNR is

$$psnr = 10\log 10 \frac{M \cdot N \cdot 255^2}{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(x(i,j)-\hat{x}(i,j))^2} \tag{2}$$

Where M, N are the number of pixels each column and row, respectively, $x(i,j)$ and $\hat{x}(i,j)$ are the gray value of original and reconstructed image at (i,j).

In the paper, the experiment image is decomposed to one, two, and three stages, respectively, and gets the values of coding gain, PSNR of each wavelet base. In those test parameters, if a wavelet base fits image compression, coding gain and PSNR should be high, and with high compact support, i.e. with the value of W_s is less.

When calculating the value of PSNR, the compressed image is reconstructed by the only coefficients of the lowest frequency subband, and makes the coefficients of other subbands zeros. It is because the human visual sensitivity changes when it is stimulated by signal with different frequency, and it is more sensitive to low frequency signal. In the progress of coding, the lowest frequency subband always is fine quantized and compressed, while the high frequency subbands are done roughly, and if the compression rate is low, they can be abandoned. Thus PSNR of the image reconstructed only by the lowest frequency band coefficients can be an evaluation standard of the wavelet base coding performance.

3. Daubechies Wavelet Bases

Daubechies compactly supported, orthogonal wavelet bases (db$N$, N is the rank of wavelet base) are most widely used, and with the properties of orthogonality, regularity, vanishing moment, they are chosen as the research objects. In addition, chosen wavelet bases of one family are convenient for analyzing contrastively, and the results have high reliability.

In the practical operation, as for the support width of wavelet base is related to the speed of wavelet transform, it is often not above 20, so db1～db10 are chosen. Tab.1 presents the properties parameter of db1~db10.

Table 1. The property parameters of wavelet bases

Wavelet bases	Regularity	Vanishing moment	Support width(W_s)
db1	discontinuous	1	1
db2	1.00	2	3
db3	2.75	3	5
db4	5.10	4	7
db5	7.98	5	9
db6	11.33	6	11
db7	15.11	7	13
db8	19.32	8	15
db9	23.95	9	17
db10	29.02	10	19

From data in Tab.1, it can be seen that the regularity and vanishing moment of Daubechies wavelet bases increase with the rank of wavelet bases growing, but the compact support decreases.

Tab.2 give the results of one, two and three stages decomposition, in the tables. According to these data and the feature parameters of wavelet bases, the correlation between the wavelet base feature and image compression can be obtained.

Tab.2 The results of experiment of db1~db10

Wavelet bases	one-stage decomposition		one-stage decomposition		one-stage decomposition	
	Coding gain	PSNR	Coding gain	PSNR	Coding gain	PSNR
db1	7.00	27.17	9.51	23.18	11.61	20.06
db2	9.45	29.07	12.64	24.64	15.85	21.50
db3	10.43	29.87	13.34	25.20	15.59	21.54
db4	11.39	30.32	14.72	25.39	18.11	22.00
db5	11.58	30.52	14.52	25.41	17.46	21.85
db6	11.31	30.47	14.80	25.52	17.86	22.08
db7	10.95	30.31	15.19	25.55	18.53	21.99
db8	10.85	30.23	14.87	25.54	17.37	22.04
db9	11.10	30.30	15.18	25.60	17.99	22.06
db10	11.67	30.49	15.28	25.63	16.78	21.87

4. The Correlation between the Wavelet Base Properties and Image Compression

4.1. Orthogonality and Biorthogonality

From the point of view of orthogonality, wavelet bases can be classified into two types: one is orthogonal and the other is biorthogonal. The lowpass filter (h) and highpass filter (g) of orthogonal wavelet base as in [4, 6] are orthogonal. Most of the orthogonal wavelet bases have infinite support, and the filters (h and g) are infinite impulse response, then the wavelet transform can't be implemented on computer. When image is decomposed or reconstructed, Daubechies orthogonal wavelet bases with finite support often be chosen.

The biorthogonal wavelet base as in [7] is composed of two wavelet functions ($\psi(t)$ and its couple wavelet $\tilde{\psi}(t)$). In the biorthogonal set, the lowpass decomposition filter (h) and highpass reconstruction filter ($\tilde{g}$) are orthogonal, and the lowpass reconstruction filter ($\tilde{h}$) and highpass decomposition filter (g) are orthogonal, too. In those filters, h and g are the filters of decomposition wavelet $\psi(t)$, while $\tilde{h}$ and $\tilde{g}$ are of reconstruction wavelet $\tilde{\psi}(t)$.

4.2. Regularity

The regularity as in [8] of function can be defined as: assuming $0<\alpha<1$, for $t,\beta\in R$, satisfying $|\psi(t+\beta)-\psi(t)|<c|\beta|^\alpha$ (c is constant), then the number of regularity of $\psi(t)$ is α. If N times differential coefficient of $\psi(t)$ also satisfies

$|\psi(t+\beta)-\psi(t)|<c|\beta|^{\alpha}$, and $r=N+\alpha$, the number of regularity of $\psi(t)$ is *r*. For wavelet function $\psi(t)$, the regularity of $\psi(t)$ can represent the effect of data compression.

From the data in Tab.1~2, it can be seen that coding gain changes much with regularity growing, but the trend is rising, and the more the decomposition stage is, the higher coding gain is, while PSNR decreases as the decomposition stage increases. PSNR changes slowly with regularity, when it is low, PSNR increases rapidly with it increasing, and becomes evenness, i.e. the regularity of chosen wavelet base for image compression needn't to be high, and less than 10 is suitable.

4.3. Vanishing Moment

If wavelet function satisfies $\int_{-\infty}^{+\infty} t^k \psi(t)dt = 0$, and *k*=0,1,2,…, *N*-1, i.e. $\psi(t)$ has *k* zeros, they imply that the wavelet has *N* vanishing moments as in [8]. In addition, assuming $\{\psi_{j,k}(t)\}_{j,k\in z}$ are the standard orthogonal basis of L^2(R), and the wavelet function $\psi(t)$ satisfies $\psi(t)=C^{N+1}(R)$, $\int_{-\infty}^{+\infty}|t|^{N+1}|\psi(t)|dt<+\infty$, then the wavelet has *N* vanishing moments. The more vanishing moments the wavelet has, the better the smoothness and localization performance of wavelet bases are.

From the data in Tab.1~2, it can be seen that the correlation between vanishing moment and image compression is similar to regularity. Coding gain and PSNR increases with vanishing moment increasing when the value is less than 4, and becomes evenness, so in image compression, the wavelet base vanishing moment is about 4.

4.4. Compact Support

Compact Support as in [4] is another important property of wavelet base, and the value is directly related to the wavelet transform speed. If wavelet $\psi(t)$ satisfies $\psi(t)=0, t\notin[a,b]$, the wavelet function is compactly supported at [*a, b*], and it has the property of compact support, thus wavelet base is compactly supported wavelet base, and the size of [*a, b*] is the support width of wavelet base. The less the support width is, the better the compact support is, and the operation quantity of wavelet transform is less.

From the data, it can be seen that the correlation between support width and image compression is similarly to vanishing moment, and it is an important reference item of wavelet base choice, especially in real time image compression.

4.5. Symmetry

If Fourier transforms of $\psi(t)\in L^2(R)$ follows $\widetilde{\psi}(\omega)=\pm|\widetilde{\psi}(\omega)|e^{-i\alpha\omega}$ (*α* is a real constant , "±" is independent of *ω*), $\psi(t)$ has linear phase. And if $\widetilde{\psi}(\omega)=\phi(\omega)e^{-i(a\omega+b)}$, $\phi(\omega)$ is a real-valued function, *a*, *b* are constants), $\psi(t)$ has generalized linear phase. If $\psi(t)$ is a real-valued function, and $\psi(\alpha+t)=\psi(\alpha-t)$, $\psi(t)$ has symmetry, otherwise $\psi(\alpha+t)=-\psi(\alpha-t)$, $\psi(t)$ has antisymmetry, i.e. if the phase of real-valued function is symmetry or antisymmetry, the function has generalized linear phase at least. The human visual system is more sensitive to symmetrical error on the edge than to dissymmetrical error, so the wavelet base with linear phase is more used in image compression.

Symmetry is another property of wavelet base. In all the orthogonal wavelet bases, db1 is Haar wavelet base, and it is symmetrical, others are not. In order to research the correlation between symmetry and image compression, D5/3 and D9/7 which are symmetrical wavelet bases are chosen to compare with dissymmetrical ones. D5/3 and D9/7 belong to Daubechies biothogonal wavelet bases, and their properties are close to those of db2 and db4 respectively, the only difference is symmetrical or not. Tab.3 represents the property parameters of D5/3 and D9/7, in order to compare distinctly, the parameters of db2 and db4 are also shown in Tab.3.

Tab.3 The property parameters of symmetrical and dissymmetrical wavelet bases

Wavelet bases		Number of Regularity	Vanishing moment	Support width
D5/3	D-wavelet	0.0	2	5
	R-wavelet	1.0	2	3
D9/7	D-wavelet	1.1	4	9
	R-wavelet	1.7	4	7
db2		1.0	2	3
db4		5.1	4	7

In Tab.3, D-wavelet refers to decomposition wavelet, while R-wavelet refers to reconstruction wavelet. From the data, it can be seen that the regularity, vanishing moment and support width of D5/3 are similar to the features of db2, and the regularity of db4 is higher than that of D9/7, but the others differ a little, so the results can be used to scale the correlation between symmetry and image compression.Tab.4 gives the results of three stages decomposition.

Table 4. The results of three-stage decomposition in Table.3

Wavelet bases	D5/3	D9/7	db2	db4
Coding gain	20.17	19.29	15.85	18.11
PSNR	22.10	22.32	21.50	22.00

From the results of D5/3 and db2, it can be seen that the coding gain and PSNR of D5/3 is better than db2, PSNR of D5/3 is higher a litter, and the support width is close to that of db2. Compared the results of D9/7 with those of db4, the difference is less than the group of D5/3 and db2, and D9/7 is better than db4.

5. Experiments and Results Analysis

In order to analyze synthetical performance of wavelet bases, we give the images reconstructed by the only coefficients of the lowest frequency band, after three stages decomposition of db1~db5, D5/3 and D9/7 in Fig.1, just because that PSNR of reconstructed image compressed by db6~db10 is close to the value of db5, and needn't be shown one by one.

It can be seen that the reconstructed image of db1 (Fig.1 (b)) has block effect visibly, and the one of db2 (Fig.1 (c)) is better, but it also has negative influence on image visual quality. The block effect in db3~db5 (Fig.1 (d)~(f)) is weaken gradually. The quality of compressed image of db5 is similar to the image of db4, then it can be deduced that the quality of images of db6~db10 are also similar to db4. Considering the quality of reconstructed image and the other standards, wavelet base db4 is better than other Daubieches wavelet bases.

The quality of images (g) and (h) are similar to the compressed image of db4. The conclusion can be obtained that the symmetrical wavelet base is better than dissymmetrical one in image compression, but the difference is not great, i.e. the symmetry of wavelet base is not the necessary condition in image compression.

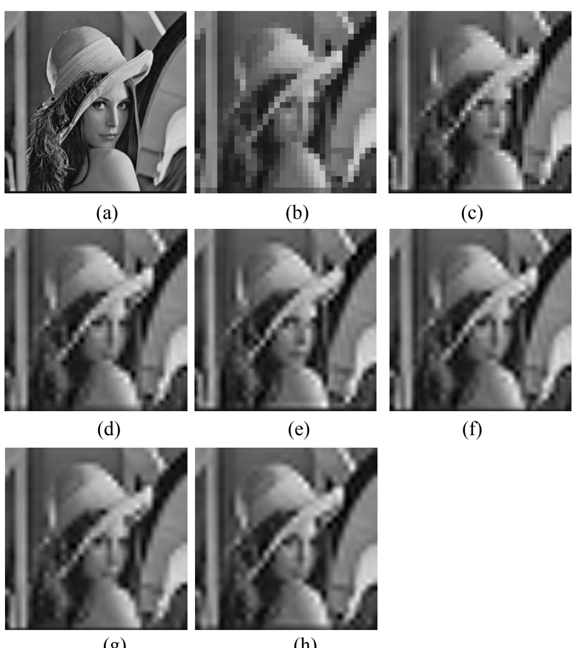

Fig. 1. The reconstructed images, (a) is the original image, (b) ~ (f) are the compressed images of db1~db5, (g) and (h) are the results of D5/3 and D9/7 respectively.

6. Conclusions

After analyzing the influence of the wavelet base properties on image compression, the conclusions can be got as follows:

- ✧ The regularity influences image compression greatly, and the regularity of chosen wavelet base for image compression is less than 10.
- ✧ The correlation between vanishing moment and image compression is similar to regularity, in image compression, the wavelet base vanishing moment should no more than 4;
- ✧ The compact support has tight correlation with transform speed, the better it is, the less time is used, so the support width should be considered, when wavelet base is chosen, especially in real time image compression;
- ✧ The symmetry has a certain influence on image compression, but it is not the necessary condition, dissymmetrical wavelet bases also can be used in image compression.

The properties of wavelet base have tight correlation with image compression, when the wavelet base is chosen, its property should be considered, and otherwise it will get half the result with twice the effort. This paper analyzed the correlation between the wavelet base properties and image compression through extensive experiments, and got the principles of wavelet base choice in image compression.

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8. References

[1] E.A.B. Da Silva, M. Ghanbari. On the performance of linear phase wavelet transforms in low bit-rate image coding. *Journal*, IEEE Transactions on Image Processing, 1996, 5(5):689-705.

[2] J. Azpiroz-Leehan, J.-F. Lerallut, Selection of biorthogonal filters for image compression of MR images using wavelet base. *Journal*, Medical Engineering & Physics, 2000(22): 335-343.

[3] Reinhard Hochmuth, Wavelet characterizations for anisotropic besov spaces. *Journal*, Applied and Computational Harmonic Analysis, 2002(12):179-208

[4] Ingrid Daubechies, Orthonoral bases of compactly supported wavelets, *Journal*, Comm. Pure Appl. Math. , 1988(41), 909-966.

[5] M. Antonini, M. Barlaud, P. Mathieu, and I. Daubechies. Image coding using wavelet transform. *Journal*, IEEE Trans. Image Processing, 1992, 1(4): 719-746.

[6] Eduardo Morales, Frank Y. Shih, Wavelet coefficients clustering using morphological operations and pruned quadtrees. *Journal*, Pattern Recognition 2000(33), 1611-1620

[7] Lowell L.Winger, Anastasios N., Venetsanopoulos, Biorthogonal nearly coiflet wavelet for image compression. *Journal*, Signal Processing: Image Commucation, 2001(16), 859-869.

[8] KE Li, HUANG Lian-qing, Choice of waveletbase in real-time compression for remote sensing image. *Journal*, OPTICAL TECHNIQUE, 2005, 36(1), 77-83